N64-18108 Code 1

TR-35(U)

FINAL REPORT - VOLUME II

Contract No. NAS5-3142

November 1, 1963

RSI SATELLITE COMMAND ANTENNA ARRAY MODEL 071-024

VOLUME II - MECHANICAL

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1.0 INTRODUCTION

This volume which forms the second part of the final report discusses the mechanical design of the Satellite Command Antenna Arrays; Model 071-024, serial numbers 1 through 4. These antennas were delivered under Contract NAS5-3142. The four major areas to be treated in this report are.

<u>Item</u>	Section	<u>Description</u>
. 1 .	3.0	Maximum driving torque
2	4.0	Maximum overturn moment.
3	5, 0	Structural integrity
4	6.0	Deflection and natural frequency

Items 1 and 2 were evaluated to insure that the array and pedestal were compatible. The investigation of Item 3 was required to minimize the possibility of any mechanical failures. Item 4 consists of two parts which are evaluated together. The deflection of the major components must be determined to insure that they are not of such a magnitude as to cause the system to be inoperative. The lowest natural frequency of the antenna must be examined to determine that it is out of the range of any expected oscillating live loads. The lowest natural frequency must be sufficiently high so that it will not be excited by the servo drive of the pedestal system. If energy is coupled into the system at its natural frequency a resonant mode may increase the stresses within the structure to a point sufficient to produce failure.

The report will discuss each of the above listed areas in the order in which they appear. However, before these calculations are discussed the defining conditions will be listed. These conditions are described in Section 2.0 which also contains a list of the symbols used in this report.

Section 7.0, the concluding section of the report, will summarize the results of the computations in Sections 3.0 through 6.0.

2.0 <u>CONDITIONS AND CONSTANTS</u>

This section of the report outlines the conditions constants and symbols used in the calculations of all the following sections. All factors are referenced to the Y axis (Figure 1) of the driving pedestal.

The following are the design goals and assumptions used in establishing design criteria.

1) Maximum acceleration - 15 degrees/sec/sec

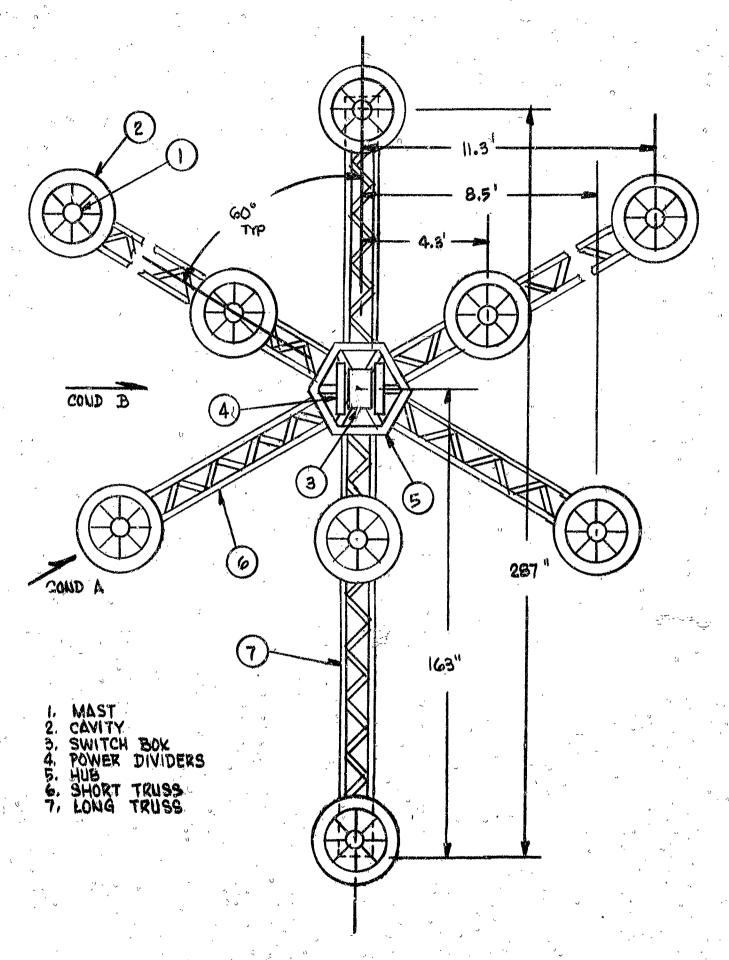


Figure 1. Antenna Array, Plan View

- 2) Maximum deceleration 150 degrees/sec/sec
- 3) Maximum radial ice load, operational 1/4 inch
- 4) Maximum radial ice load, stowed 1/2 inch
- 5) Maximum angular velocity negligible
- 6) Maximum wind velocity, operational 45 mph
- 7) Maximum wind velocity, stowed 100 mph
- 8) Stowed position zenith
- 9) Maximum available torque 25,000 feet pounds
- 10) Minimum natural frequency, operational 9 cps.

The following constants are used for the computations in this report.

Ice density 0.0324 lbs./cu. in. Wind pressure
45 mph 0.0356 lbs./sq. in.
100 mph 0.178 lbs./sq. in.

The material properties used are those contained in the "Alcoa Structural Handbook" published by the Aluminum Company of America, Pittsburgh, Pennsylvania.

3.0 REQUIRED DRIVING TORQUE

This section of the report discusses the driving torque required to operate the array under the conditions covered in Section 2.0. By assuming all factors to be maximum (worst case) the maximum required torque is obtained. Inspection of Figure 2 shows that it is only necessary to investigate the torque between the 0 and 90 degrees elevation limits.

The equation below may be used to determine the torque required to drive the antenna.

$$\Sigma T = Ia \tag{1}$$

This expression establishes the relationship between the resultant torque, moment of inertia and angular acceleration. The resultant torque in the present example is that due to the unbalanced array weight, the unbalanced weight of the ice load, the wind pressure and the driving torque. Thus, equation (1) may be expressed in an extended form as

$$\Sigma T = T - r (W_{sys}) - r' (W_{1Ce}) - r'' F = Ia$$
 (2)

^{1. &}quot;Alcoa Structural Handbook", published by the Aluminum Company of America, Fittsburgh, Pennsylvania.

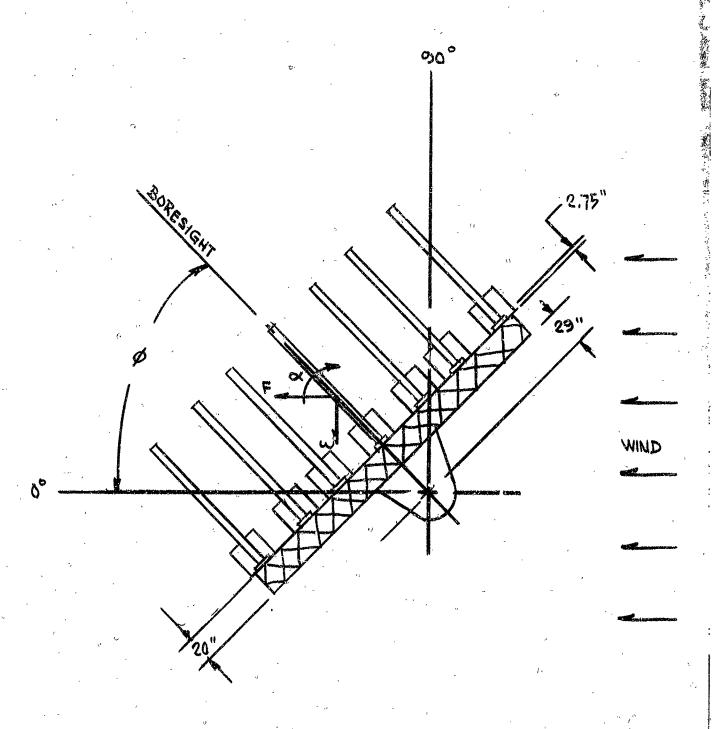


Figure 2. Antenna Array Elevation

where T is the driving torque, W is the weight, r is the respective moment arm and F is the force of the wind. All of the above items can be determined except T which is the desired unknown. Equation (2) may now be rearranged to the format:

$$T = r (W_{syst}) + r' (W_{ice}) + r'' F + I_a$$
(3)

This expression indicates that the total driving torque is the sum of the torques for each individual contributing factor. Thus, the following four sections will treat each factor separately. Section 3.5 will evaluate the total effect.

3.1 Weight Effect

This section discusses the first term of equation (3) which is the torque due to the array not being mounted at its center of gravity.

Inspection of Figure 2 will indicate that the torque produced by the offset weight is a function of the elevation angle, β (ie: the moment arm will vary with the elevation angle). The weight torque can be expressed as

$$r (W_{SYS}) \cos \theta$$
 (4).

where θ is the angle the moment arm makes with the horizontal. Examination of Figure 1 will show that the center of gravity will be located along the boresight axis since this is an axis of symmetry. Thus the angle θ is equal to \emptyset and r is the distances from the y axis to the center of gravity measured along the boresight axis. Equation (4) thus becomes:

$$r (W_{SYS}) \cos \emptyset$$
. (5)

If the above expression is used in conjunction with Table A then Equation (5) may be altered to read

(6).

3. 2 Ice Loading Effect

The object of this section is to discuss the second term of Equation (3) which represents the offset loading caused by the 1/4-inch radial iding. The ide load will vary in the same manner as the system weight load. The total weight and the center

TABLE A

Item	Mast	Cavity	Switch Box		Element Cables	Structure	e Total
Weight each - lbs.	95	38	41	28	3	11342	
C.G. from Y axis - ft. 1	11	4.9	4.4.3	4.,43	4.13	3.43	
Moment each - ftlbs.	1045	186	180	123	12	3850	
Quantity required	9	9	1	2	18	1	
Total weight - lbs.	855	342	41	56	54	1134	2,482
Total moment - ftlbs.	9400	1674	180	246	216	3850	15, .66
	,			,			

NOTES: 1) Based on back surface of structure at 29 inches from Y axis (Figure II-2).

- 2) Based on original weight of 1039 lbs. with the addition of 95 lbs. of strengthening members and hardware.
- 3) Estimated

 $T_{\nu} = 15,566 \cos \emptyset$

W = 2,482 lbs.

b = 6.27 ft.

of gravity must be calculated. These parameters may be computed by considering each typical component and combining the results to obtain the total loading. Table B details the ice load (per inch) for various shapes. This table will be used to determine the weight of the ice in the system. Sections 3.2.1 through Section 3.2.5 discuss the ice loading effect on each major component while Section 3.2.6 discusses the resultant total torque due to ice loading.

3.2.1 Mast (Figure 3)

The center of gravity and weight of the ice load on a typical mast are determined by summing the effects of the individual components that make up the mast. The following tabulation presents the calculated weights and the location of the center of gravity (from the base of the mast) of the various elements.

<u>Item</u>	Weight (lbs.)	r(in,		Moment (in. lb	<u>s.</u>)
End cap	0.10	230	1,	23	
3.5" tubing	6.46	196		1266	
3.5"-4.5" join	it 0.06	162		10	U
4.5" tubing	12, 15	112		1360	
4.5"-6.0" join	nt 0.11	61			
6.0" tubing	9, 50	31	••	244	··
Dipoles	5, 51	22	2 1	121	
Flange	. 46	,,75		1	
Disks	39, 40	130		5102	-
. 1) 	72,65	NA.	,	8164	

The total weight of the ice load on one mast is thus 72,65 pounds. The effective center of gravity is determined by the moment equation

$$r = \frac{\Sigma M}{\Sigma W}$$
 or $\frac{8184}{72.65} = 112.5 in.$ (7)

Since the moment arm must be relative to the Y, axis, the displacement of the mast base must be added to Equation (7) and the result expressed in feet.

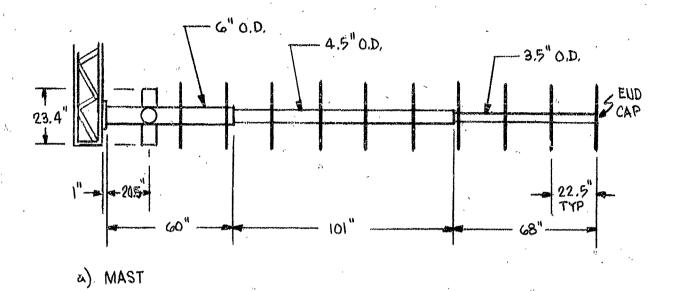
Results:

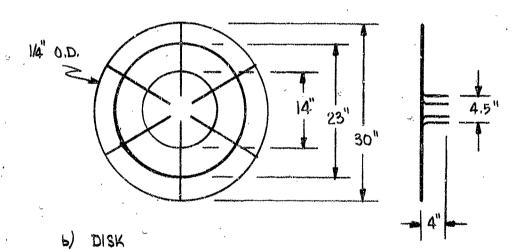
$$W = 72.65 \text{ lbs.} \approx 73 \text{ lbs.}$$

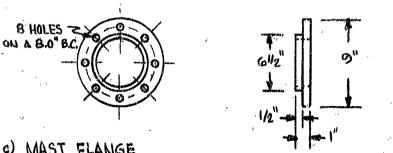
 $r = 13.7 \text{ ft.}$

3. 2. 2 Cavity (Figure 4)

The procedure of Section 3.2. I was used to compute the moments for the

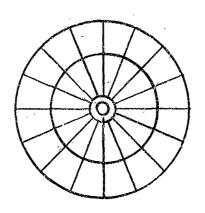






c) MAST FLANGE

Figure 3. Mast



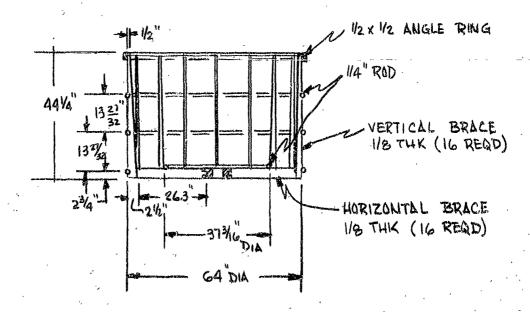


Figure 4. Cavity

cavity members. Their moments are listed below.

<u>Item</u>	Weight (lbs.)	<u>r (in.)</u>	Moment (in-lbs.)
Large dia. rings (3)	7,68	16.59	127.0
Small dia, ring	1.48	2.75	4.1
$1/2 \times 1/2$ angle ring	4.85	44, 25	214.3
Vertical brace	24.80	17.20	426.0
Horizontal brace	20,40	13.75	<u> </u>
Totals	59, 21	NA	799.4

The total weight of the ice load is thus 59.21 pounds and the center of gravity, relative to the cavity base, is

$$\frac{799.4}{59.21}$$
 = 13.5 in. .

The location of the center of gravity of the ice load expressed in feet and referenced to the Y axis is then 5.4 feet.

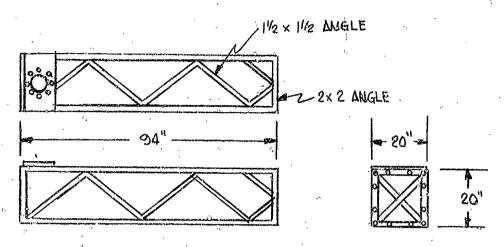
Results:

W = 59. 21 lbs,
$$\approx$$
 59 lbs., $r = 5.4$ ft.

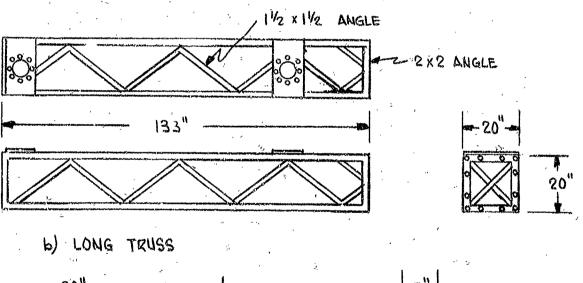
3, 2, 3 Structure (Figure 5)

The weight of the ice on the structure was determined by calculating the lengths of the various types of structural materials used and multiplying this by the appropriate weight per length data presented in Table B. The center of gravity of the ice load on the structure is assumed to be the same as that of the structure, or 3.4 feet. The following tabulation presents the data used to calculate the weight of the ice on the structure.

<u>Item</u>	Length (in.)	W/1 (lbs./in.	W(lbs.)
$1-1/2 \times 1-1/2$ angle	4698	. 057	267.5
2 x 2 angle	4031	.073	294.0
4 x 4 angle	768	. 138	105.0
2" dia, rod	115	. 057	6.6
I beam	. 197	. 125	24.6
Channel	1,34	, 103	13.8
	Oty. (ea.)	W/ea, (lbs/ea.)	W(lbs.)
Mast mtg. plate	9	5, 31	47.8
, ,		Total	760.3



a) SHORT TRUSS



20"

5" DIA

14" PLATE

11/2 x 11/2 ANGLE

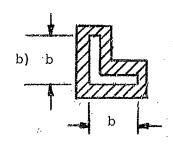
16"

Winnes 5 Transact

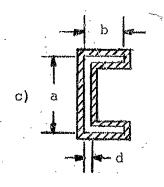
TABLE B Ice Loads per inch (ice density = $.0324 \text{ lbs/in.}^3$)

a)	D			
$\frac{wt}{\mathcal{I}}$	= πt (D+t) ρ	-	ŝ	,

D(in.)	lbs/in.t = 1/4"
.25	.0127
2,0	. 0571
3.5	. 0951
4.5	.1205
6.0	. 1585
6.5	.1710
9.0	.234



$$\frac{\text{wt}}{\ell} = 4 (t^2 + tb) \rho$$



Results:

$$W = 760 \text{ lbs.}$$

r = 3.4 ft.

3.2.4 Switch Box (Figure 6)

The ice load on the switch box was calculated by assuming all sides of the box, except the bottom, to be covered with 1/4 ice. The ice load on the under side is neglected since it rests on the structure.

Results:

$$W = 11 lbs.$$

r = 4.4 ft.

3.2.5 Power Dividers (Figure 6)

The power dividers, consisting of two packages of four elements, is shown in Figure 6(b). The weight of the ice load for one element is calculated to be 4.34 pounds. Thus, each package has a total ice load of 17.36 pounds. The center of gravity of the ice load is considered to be the same as that of the power dividers, or 4.4 feet from the y axis.

Results:

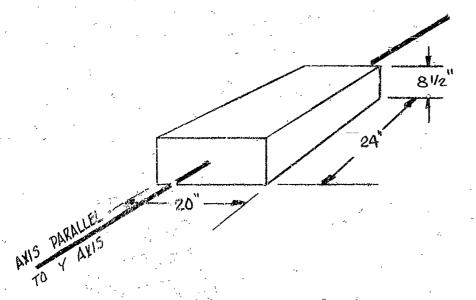
$$W = 17 \text{ lbs.}$$

r = 4.4 ft.

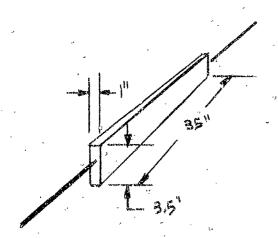
3.2.6 Total Effect of 1/4 inch Ice Loading

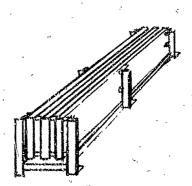
The results of the computations in Sections 3.2.1 through 3.2.5 are summarized below. The resultant torque is obtained by summing the moments due to the ice loads on each component.

<u>Item</u>	Mast	Cavity	Structure	Switch Box	Power Divider	Total
W (lbs.) ea.	73	o 59	760	11.	17	ે જે અ •્ર
c.g. (ft.) ea.	13.7	5.4	3.4	4.4	4.4	
M (ft1bs.)ea.	1000	318	2580	48	75	'
Qty.	9	9	1	1	a 2	***
W total	657	531	760	. 11	34	1993
M total	9000	2862	2580	48	150	14640



a) SWITCH BOX





L) POWER DIVIDER

Figure 6. Switch Box and Power Divider

Thus, the second term of Equation (3) is expressed in the general form

3.3 Wind Torque

This section of the report will discuss the torque required to overcome the moment caused by wind loading. The wind loading represents the third term of Equation (3). Since wind loading is a function of cross sectional area, the following calculations are based on an ice loaded system.

The basic formula used to calculate the wind leading torque is.

$$T = PArC,$$
 (8)

where P is the pressure exerted by a 45 mph wind, A is the area projected into the wind, r is the moment arm from the y axis to the center of pressure and C is the shape factor.

As with the previous weight torques the wind moment arm r, varies with the elevation angle and the center of pressure is considered to act along the boresight axis. In this case the resultant wind force is horizontal. Therefore the moment arm will vary as the sine of the elevation angle. Thus Equation (8) becomes

$$T = P \times A \times r \sin \phi \times C \tag{9}$$

The projected area and shape factor will also vary with the elevation angle. To allow for this variation the total effect is considered to be the sum of two separate forces. These forces are the forces which act on the area projected into the wind at 0 degrees elevation and the area projected at 90 degrees elevation. These forces are then considered to vary respectively as the cosine and sine of the elevation angle. Thus, Equation (9) becomes

$$T = P(A' C' r' \sin \emptyset \cos \emptyset + A'' C'' r'' \sin^2 \emptyset)$$
 (10)

where A, C and r are the factors for the respective end conditions of elevation angle (0 degrees ('): 90 degrees (")). The wind loading on the structure is assumed to be condition B as shown in Figure 1. This is the worst case.

The following table presents the calculated results for the various components based on the shape factors outlined in Table C.

TABLE C
Shape Factors for Wind Loading

<u>Item</u>	C: ~ 00	<u>C" - 90°</u>	Remarks
Mast tubing	N.A.	1	Reynolds number - 10 ⁵
Disks	1	· 1	Reynolds number - 105
Cavity rods	1	1	Reynolds number - 105
$1/2 \times 1/2$ angle	2.0	1.6	Aspect ratios of 32 and 90
Gussets	1.4	1.4	Aspect ratio of 20
Switch box	N.A.	1.17	Aspect ratio of 3
Truss	2.0	2. 0	Various approaches Yielded 1.76 to 2.0 - maximum condition selected.

-		$\emptyset = 90^{\circ}$	٠	•	
<u>Item</u>	Mast	Cavity	Structure	Switch Box	<u>Total</u>
Force (1bs.) ea.	52	102	444	4 9	
Oty.	9 .	9	1	. 1	
Force (1bs.) total Center of pressure	468	918	444	9	· m
(ft.)	13.4	5.4	″3 . 2	4.4	~ 3
Torque (ftlbs.)	6270	4950	1420	40	12680
,		ø 0°			

<u>Item</u>	Mast	Cavity	Structure	<u>Total</u>
Force, ea.	84	45	600	, ,
Qty.	. 9 ".	9	1	-
Force, total	756	405	600	. **
Center of pressure	14.2	6.3	3, 2	
Torque	10,730	2550°	1920	15200

Equation (10) may now be implemented to read

$$T = 15200 \sin \emptyset \cos \emptyset + 12680 \sin^2 \emptyset$$
. (11)

3.4 Acceleration Requirement

This section discusses the final term of Equation (3); the torque required to accelerate the antenna system. The acceleration torque is a function of the moment of inertia and the desired acceleration.

The desired acceleration, as listed in Section 2.0, is 15 degrees/sec/sec. This acceleration is equivalent to 0.262 radians/sec/sec.

The moment of inertia is determined on a component basis. This is accomplished by calculating the moment of inertia of each component about its center of gravity and then transposing to the y axis using the parallel axis formula

$$I_y = I_0 + \frac{W}{32} r^2$$
 (12)

Although the parts considered consist of ice and aluminum they are tested as homogeneous. The weights and centers of gravity are thus those of the ice loaded components.

The following tabulation presents the calculated results for the various components of the antenna system.

,	Io	Mass	r^2		$I_{\mathbf{y}}$
<u>Item</u>	(Slug-ft. ²)	/ C1	(Ft. ²)	Qty.	(Slugft. 2)
Mast	161	5, 25	148	3	2790
Mast	161	5, 25	220	2 ·	2616
Mast	161	5.25°	276	2	3204
Mast	161	5, 25	167	2	2051
Cavity	14.2	3,03	26	3	240
Cavity	14.2	3.03	98	2	594
Cavity	14.2	3.03 .	154	, 2	934
Cavity	14.2	3.03	44	ຶ 2	268
Truss	· 9, 0	19.7	11.5	1 '	237
Truss	705	19.7	13.4	2	1938
Power divid	ers 0	1.4	19.3	2 .,	55
Switch box	Ó	1.6	19.3	· 1	31
	" , "	,		Total	14958 (Slug-
,					ft. ²)

The four different values for r^2 in the above calculations for the masts and cavities are a direct result of their different offset from the y axis. The values for the trusses are based on one unit being parallel to the y axis and two units inclined at 60 degrees. Each truss unit was considered to consist of one short and one long truss and one-third the weight of the hub.

Using the above calculations the torque required for acceleration may now be expressed as

$$T = Ia = 14958 \times 0,262 = 3920 \text{ ft.} -lbs.$$
 (13)

3:5 Maximum Torque

Equation (3) may now be fully implemented to read

$$T = 15566 \cos \beta + 14640 \cos \beta + 15200 \sin \beta \cos \beta + 12680 \sin^2 \beta + 3920$$
 (14)

This expresses the required driving torque as a function of the elevation angle for an acceleration of 15 degrees/sec/sec and for 1/4 inch radial ice loading.

Figure 7 is a plot of the torque as a function of the elevation angle. The maximum torque is 40,000 foot pounds and occurs at \emptyset = 39 degrees.

4.0 OVERTURN MOMENT, STOWED

The object of this section is to compute the overturn moment about the y axis when the antenna system is in the stowed condition. One-half inch ice loading and a wind velocity of one hundred miles per hour is assumed.

In the stowed position the only force tending to overturn the array is the wind. Therefore, it is only necessary to perform calculations similar to those of Section 3.3 for ø equal 90 degrees but compensation must be included for the increased area caused by the greater ice load and for the increased pressure caused by the higher wind velocity. The following summary presents the calculated data for the three major components of the antenna system.

<u>Item</u>	Mast	Cavity	Structure	<u>Total</u>
Force (lbs.) ea.	318	. 657	2700	was
Center of pressure		, 3	,	
(ft.)ea.	12.85	5.2	3.2	,
Moment (ft1bs.)ea.	4080	3420	8640	••
Qty.	9	9	1	****
Total force	36720	30800	8640	76,160

Thus, the total overturn moment in the stowed condition is 76,160 ft.-lbs.

5. 0 STRESS ANALYSIS

This section contains the computations which define the maximum stresses in the major components of the system. The values used in the calculations are for the most severe conditions of Section 2.0. A deceleration of 150 degrees/sec/sec is assumed as an operational condition.

In order to determine whether the maximum stress is within a safe limit for a particular element, it is compared to the allowable stress by the formula

where S. F. is the resulting safety factor. The allowable stresses used in the following calculations are 13 kips/sq. in. for aluminum parts and 120 kips/sq. inch for mounting hardware. The allowable stress for aluminum in both tension and

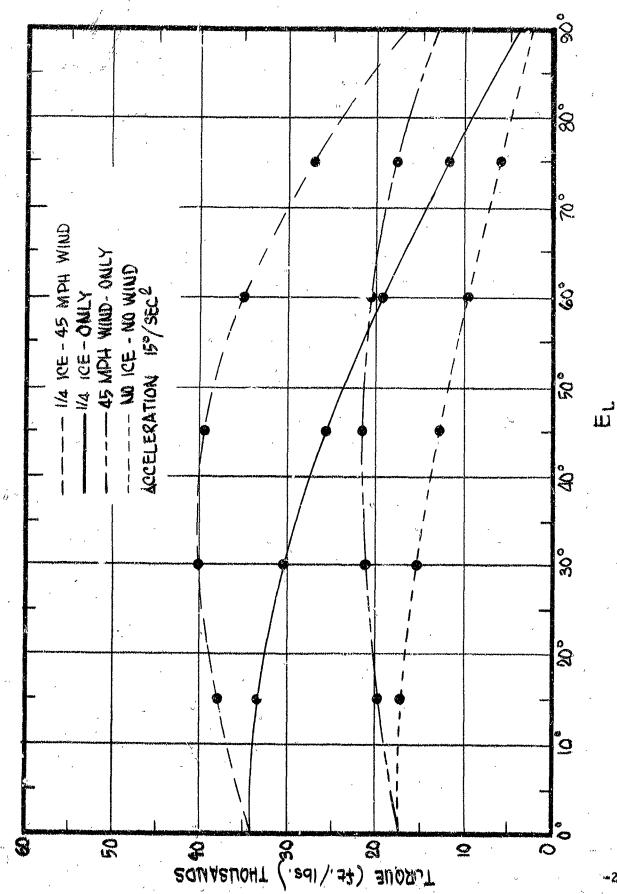


Figure 7. Torque versus elevation angle.

compression is based on a reduction of the yield strength of 6061-T6 aluminum from 35 kips/sq. in, caused by the localized heating effects of welding. The allowable stress for the mounting hardware is based on the ultimate tensile strength of SAE grade 5 hardware up to 3/4 of an inch in dismeter.

The first consideration is that of the mast under stowed conditions. In this case the stress in the extreme fiber of the tubing was considered to be due to the weight of the element and half inch ice in addition to the bending moment caused by the wind loading. Column buckling is neglected based on the assumption that the ice load is self supporting although it is not considered to assist in the stress distribution. The latter assumption results in a condition which appears more severe than the actual condition. The maximum stress in the extreme fiber was determined by the expression

$$s = \frac{W}{A} + \frac{F_{\Gamma}}{S} \tag{16}$$

where W is the total weight, A is the cross section area of the stress devicer. Fr is the bending moment due to wind loading and S is the section of the stressed member. The first term in Equation (16) expresses to extreme fiber due to axial loading while the second term provided are the joints of the different tube diameters. These are the points of the different tube diameters. These are the points of the different tube diameters. The information with and bending moments are maximum for each member. The information 4.0 was used to compute the stresses and safety factors in the table below.

Tubing	3, 5" Dia.	4. 5" Dic.	6" Dia,
Axial load (lbs.	75	178	286
Bending moment (in1bs.)	3500	20320	36625
Stress (pair)	3297	. 8880	7683
S. F.	3, 96	1.46	1,69

Thus the lowest safety factor of 1.69 is at the junction of the 4-1/2 and 6-inch mast sections.

The following evaluation is that of the mast mounting hardware during stowed conditions. In this case only the bending moment was considered because it is the only tension producing force. As shown in Figure 3 the mast is mounted by eight 5/8 inch diameter bolts. For calculating the maximum load the mast was considered to be mounted by two sets of four 5/8 inch diameter bolts at right

angles with each bolt pattern taking a vector portion of the total load depending on the direction of the wind. To assume the worst case, it was considered that one set had no load while the other bore the maximum load. This assumption was made to simplify the calculations, but of course results in a load greater than the actual. Thus, the 42, '90 in.-lbs. moment produced by the wind on the mast and cavity results in a 3740 pound load per bolt. To determine the safet factor allowance made for the initial stresses due to the setting of the bolt, two me hods were investigated. The latter indicated that the pre-stressing on this size of bolt is the greater contributing factor. The first analysis was based on the formula 1)

Working load = 120,000 (
$$.55d^2 - .25d^2$$
), (17)

where d is the diameter of the bolt and 120,000 is the allowable stress. From this equation then the allowable load for a 5/8 inch bolt is 7200 pounds which provides a safety factor of 1.92. The second approach was to use the pre-stress factor of 16,000 pounds per inch of bolt diameter 2). This provides an initial load of 19,000 pounds which produces a total stress of 83,800 pounds per square inch on the bolt. The safety factor based on this approach is then 1.76.

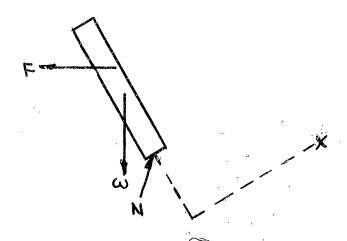
The next consideration is the mast under operational conditions. As in the stowed condition, the maximum stress is due to both the axial and transverse loads which in this case includes the axial load due to translation and the bending moment due to rotational deceleration as shown in Figure 8. Also, as before, the points considered are the weldments of the tubes of different diameters. The expression used for the total stress is

$$s = \frac{F \cos \emptyset + W \sin \emptyset + W/32 \text{ ar}}{A} + \frac{Ia}{S}$$
 (18)

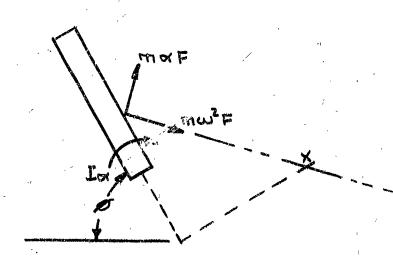
where F is the force due to the wind, W is the total weight, a is the angular acceleration, r is the distances from the y axis to the c.g. and I is the moment of inertia about the point being considered. The following summary presents the calculated data used in Equation (18) to arrive at the maximum stress for the three

^{1.} Erik Oberg and F. D. Jones, "Machinery's Handbook", 1946.

^{2.} Lionel S. Marks, "Mechanical Engineers' Handbook", McGraw-Hill Book Company, Inc., 1941.



A) EXTERNAL FORCES



b) REACTION FORCES

Figure 8. Free Body Diagram, Mast

individual tube diameters.

Tubing Size

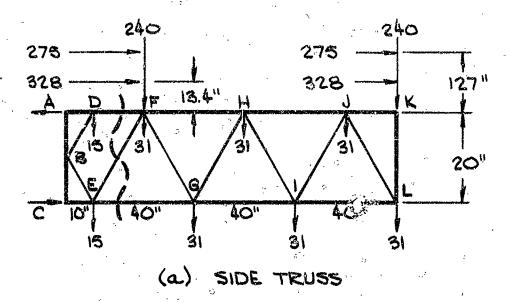
Parameter	3.5" dia.	4,5" dia.	6" dia.
W (lbs.)	37.2	96.4	168
F (lbs.) 20.4 s	in $\emptyset + 30.6 \cos \emptyset$	49 sin Ø + 61 cos Ø	72 $\sin \emptyset + 84 \cos \emptyset$
W/32 ar		•	,
(lbs.)	34.4	u	156
Max. axial load (lbs.	86	225	355
I (slug-ft. ²)	14.1	235	640
Max. stress	476	2899	4304
S.F.	27.3	4.5	3.02

The minimum safety factor in this instance occurs at the base of the mast.

Next, consideration is given to the mast mounting hardware under operational conditions. Using the same approach as in the stowed condition the load per bolt was calculated to be 1800 pounds. This load is the result of bending moments exerted by the wind on the mast and cavity. Superimposed on the stress due to transvers loading is a 355 pound axial load which is divided equally between the eight bolts. The total resultant load is 1890 pounds per bolt. The working load of 7200 pounds, as determined by Equation (17), is used, a safety factor of 3.8 is realized. If the previously estimated setting load of 10,000 pounds is used, the safety factor may be computed as 2.0.

In order to determine the safety factor for the truss mounting hardware, under stowed conditions the truss was assumed to be mounted by only the top and bottom rows of bolts. Each row has four bolts which are 5/8 of an inch in diameter. The longer truss, which supports two masts; was selected for evaluation. Using the half loads shown in Figure 9 and taking the moments about point C, the load per bolt was calculated to 3877 pounds. If allowances are made for the initial bolt stress, the safety factor may be computed as either 1.9 or 1.8 depending on which bolt setting criterion is used.

The final item to be evaluated is the truss. The longer truss is selected for analysis. Figure 9 shows the assumed loading under stowed conditions for one side of the truss. It is assumed that the two sides share the total load equally and that the internal members have no load. By inspection of Figure 9 it is apparent that the highest stressed members are either those cut by the section line or those to the left of the section line. The members AD, BD, BE and CE form a statically indeterminate system. Hence starting at the section line simplifies the problem. This approach is demonstrated in Figure 9(b) where the sum of the forces in any direction must be zero. With these three equations and the three unknowns the



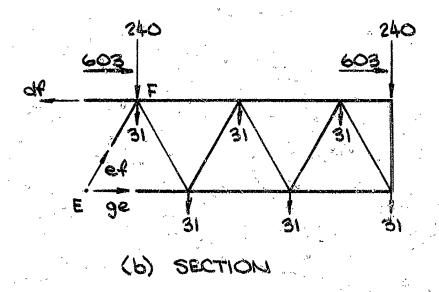
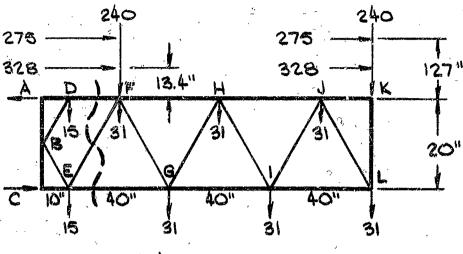


Figure 9. Truss Loading Diagram (Stowed)

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(a) SIDE TRUSS

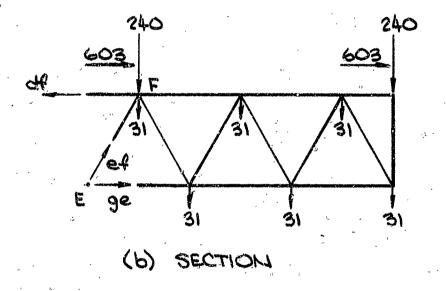


Figure 9. Truss Loading Diagram (Stowed)

force in each member can be determined. Using this technique the following data was obtained.

<u>Member</u>	Force (lbs.)	Type of Angle
DF	3537 (T)	2 x 2 x 1/4
GE	1665(C)	$2 \times 2 \times 1/4$
EF	944(C)	$1-1/2 \times 1-1/2 \times 3/16$

Having obtained this data the forces on the remaining members to the left of the section line were determined by the method of joints for a plane truss. This results in the highest loaded members of each type of angle being BE and AD. The following table describes the results obtained.

<u>Item</u>	2 x 2 x 1/4 Angle	$1.5 \times 1.5 \times 3/16$ Angle
Member	AD	BE
Load (1bs.)	3552 (t)	964(t)
Area (sq. in.)	. 94	. 53
Stress (psi)	3780	1820
S.F.	3.44	7.15

The safety factor for buckling is greater since the welded joints are considered only to effect the end pin conditions of the angle columns. Using an end factor of 0.75, "effective slenderness ratios" of 77 and 72.4 were obtained. These ratios were used to predict column strengths of 17.2 and 19.7 kips/sq. in. Since these allowed buckling stresses are greater than the 13 kip/sq. in. allowed for the welded joints it follows that the safety factor in buckling is also greater.

6.0 <u>DEFLECTION AND NATURAL FREQUENCY</u>

This section of the report discusses the maximum defloction and natural frequency of the mast and backup structure. The deflection (δ) and natural frequency (f) are treated together because the natural frequency is determined from the deflection as shown in the following formula for a cantilevered beam

$$f = 3.89/6^{1/2}$$
 (19)

The lowest natural frequency is the one of interest, hence the maximum deflection must be used. Also, since the natural frequency is only of concern when used with the pedestal the computations will be limited to operational conditions only. The following equations to determine the natural frequency of the various mast components were used

Uniform Load: $\delta = (W + F) L^3/8EI$;

Concentrated Load at h:
$$\delta = (W + F) h^2 (3L-h)/6EI;$$
 (21)

where W is the weight of the element, F is the force of the wind, L is the length of the member, E is the modules of elasticity, I is the moment of inertia of the cross sectional area of the member and h is the distance from the mounting point to the concentrated load. The condition assumed to be the worst case was with a 45 mph wind but no ice. This assumption was made on the basis that, although the weight would be increased by the weight of the ice, E and I would be increased to an extent that the resulting deflection would be less. Furthermore, even if the ice loading did slightly lower the natural frequency, resultant vibrations would cause the ice to drop from the structure thereby raising the resonant frequency again. Using equations (19) (20) and (21) the following results were obtained

Tubing Sizes	3,5" dia,	4.5" dia.	6" diz.
Uniform load (1bs.)	17.4	41,3	60.6
Concentrated load	2:2	2.2	2,2
(1bs.)			, ' '
at $h = (in,)$	68,46,23 and	10 11,34,56	27 and 49
•		and 79	
Concentrated load	. 0	26.1	75.2
(1bs.)		0	
at h = L (in	.) 0	101	60 •
δ (in.)	0,06	0.258	0.056
f (cps.)	15,8	7,65	16.4

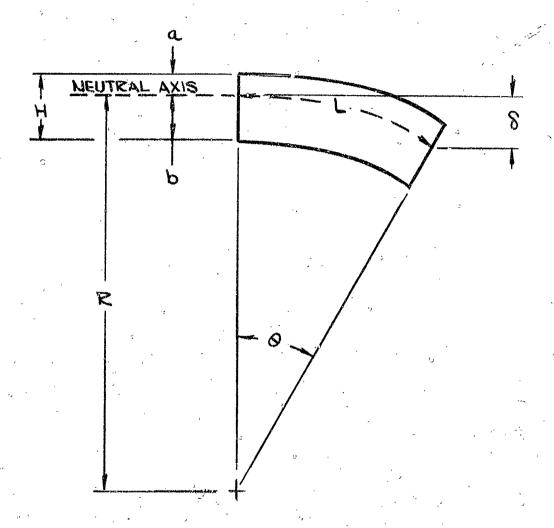
The deflection of the truss under operating conditions was determined by the formula

$$\delta = \frac{L}{2} \left(\frac{\Delta L_{r} + \Delta L_{C}}{H} \right) \tag{22}$$

where L is the length of the truss, ΔL_r and ΔL_c are the changes in length of the tension and compression members respectively and H is the height of the truss. The derivation of this formula is shown in Figure 10. The total change in length of the two members was calculated by summing the changes in lengths for each member shown in Figure 11. This figure shows the load for each side of the truss under operational conditions. The change in length for each member in turn is based on the stress strain relationship

$$Z_{\lambda} = \frac{FL}{AE} \tag{23}$$

where E is the modulus of elasticity, F the load, A the area, L the initial length and Δ the change in length. The following tabulation presents the calculated



$$8 \cdot R - R \cos \theta$$

$$R\theta \cdot L$$

$$8 \cdot \frac{L}{\Theta} (1 - \cos \theta)$$

$$1 - \cos \theta \cdot 1 - 1 + \frac{\theta^2}{21} - \frac{\theta^4}{41}$$

$$8 \cdot \frac{L}{\Theta} (\frac{\theta^2}{2} - \frac{\theta^4}{41})$$

$$6 \cdot \frac{L}{\Theta} \frac{\theta^2}{2} - \frac{\theta^4}{41}$$

Figure 10. Truss Deflection

$$(R+\omega)\Theta=R+\Delta L_T$$
 $(R-b)\Theta=R-\Delta L_C$
 $(\omega+b)\Theta=\Delta L_T+\Delta L_C$
 $\Theta=\frac{\Delta L_T+\Delta L_C}{H}$

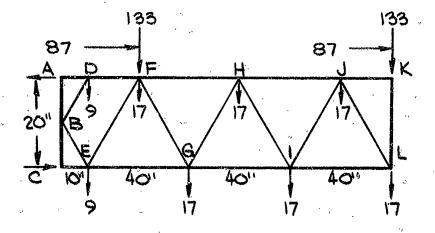


Figure 11. Truss Loading Diagram (Operational)

loads on each member and the resulting changes in length,

•	(1)	(2)		
<u>Member</u>	<u>L</u>	Load	$(1) \times (2)$	$\Delta \times 10^{-4}$
AD	10	1471	14710	15.2
DF	20	1462	29240	30.2
PH	40	789	31560	32,6
HJ	40	404	16160	16.7
JK	20	87	1740	1.8
CE ,	10	1665	1.6650	17.2
EG	40	920	₂ 36800	38.0
GI	40	501	20040	20,7
IL	40	150	6000	6.0
			T	otal $178, 4 \times 10^{-4}$

Thus, from equation (22)

$$\delta = \frac{130}{2} \left(\frac{178.4 \times 10^{-4}}{20} \right) = .0583 \text{ in},$$

Equation (19) may now be used to calculate a natural frequency of 16.7 cps.

7.0 CONCLUSIONS

This section will discuss the results obtained in the previous sections relative to the design goals set forth in Section 2.0. Table D presents a summary of the data obtained in addition to some data not present in this report. This latter data was tabulated using the procedures described in this report. Prior to discussing the individual results, it must be remembered that all calculations were for the worst practical conditions so that in each quantity expressed there is some additional safety factor included.

The first item to be discussed is the required driving torque. The following maximum driving torques were computed in this report:

<u>Condition</u> <u>Ma</u>	x. Torque (ft. lbs.)	Pos	ition
No ice, no wind, maximum acceleration:	17,776	ø	0 ^O .
No ice, 45 mph wind, maximum acceleration:	22,000	Ø	45 ⁰
1/4" ice, no wind, maximum acceleration:	34,126	Ø	00
1/4" ice, 45 mph wind, maximum acceleratio	n: 40, 000	ø	39 ⁰

Square law curves may be used to interpolate for driving torques for different icing and wind loading conditions. It should be noted that the above conditions are for the worst possible cases. While the cases are theoretically possible, they are impractical from an operational viewpoint. This is simply because the time the system will spend under these adverse conditions is essentially negligible. An average or RMS driving torque would probably be a more appropriate number to use. In any event it would appear that the mount would drive the antenna even under the most adverse conditions if some of the ice loading is manually removed.

The only marginal resonant frequency that has been computed occurs in the mast with maximum wind loading. This resonance becomes possible only when the wind is broadside to the disc-on-rod elements and has a velocity of 45 mph. For head-on winds or no winds the minimum resonant frequency of the mast is 11 cps. This resonant condition could be encountered during an operational period. The question, of course, is the definition of that damage which might occur within the system. The element safety factors are quite high and it is therefore not expected they will fail. The truss is overdesigned and, using pessimistic load figures, has been found to have a minimum resonance of 16.7 cps. Since this structure constitutes about 50 percent of the system mass, it is also quite unlikely that the drive pedestal will be significantly affected by the resonance. (i.e., The system resonance lies somewhere between 16.7 and 7.4 cps.) Consequently, it is felt that the above-noted marginal resonance in a "worst condition" environment will not significantly impair the system's operation.

TABLE D

1/4 Inch Ice Load - 15 degree/sec/sec acceleration

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Weight Center of gravity from y axis Required driving torque	s 45 mph wind No wind	4475 lbs, 6.76 ft. 40,000 ftlbs 34,126 ftlbs
1/2 Inch Ice Load - 100 mph	wind - stowed	,
Weight Center of gravity from y axis Overturn moment about y axis		7372 lbs. 7.1 ft. 76,160 ftlbs
No Ice - 15 degree/sec/sec A	<u>acceleration</u>	•
Weight Center of gravity from y axis Required driving torque	s 45 mph wind No wind	2482 lbs. 6.27 ft. 22,000 ftlbs. 17,746 ft,-lbs.
Minimum Natural Frequency	, e	
Mast Truss	45 mph wind No wind	7.7 cps. 11.0 cps. 16.7 cps.
Minimum Safety Factors	0	
Mast - stowed Mast - operational Mast mounting hardware - sto Truss - stowed Truss mounting hardware - sto Mast Mounting Hardware - op	pwed	1.46 3.0 1.76 3.44 1.8 2.0

The final area to be discussed is that of safety factors all of which appear satisfactory but a few deserve some comment. The 1.46 S.F. shown for the stowed mast occurrs at the junction of the 6.0 inch and 4.5 inch spars. In the calculations, the effect of the ice in bearing some of the load was neglected and not only might this be a considerable factor, it is probably more realistic to assume that in the stowed condition the ice at the base has a greater cross sectional area than that above it. Also, it should be remembered that this is a compressive load and that the tension stress is less. The next two lowest safety factors are also for the stowed condition, but are for mounting hardware. In both of these cases all of the hardware was not considered to support the total loads and in addition the load was considered to be the total of the initial load and the working load with the initial load being the greater. Also, an alternate method of calculation shows the mast mounting hardware under the stowed conditions to have a safety factor of 1.92 rather than 1.76. As can be noted, the minimum safety factors occur under the stowed conditions. Thus, if there is any concern it might be reasonable to provide guy lines in the stowed condition whenever there is any possibility of exceeding the design requirements of 100 mph wind velocities.